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Supersymmetry of gravitational ground states

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Abstract: A class of black objects which are solutions of pure gravity with negative cosmological constant are classified through the mapping between the Killing spinors of the ground state and those of the transverse section. It is shown that these geometries must have transverse sections of constant curvature for spacetime dimensions d below seven. For $d \geq 7$, the transverse sections can also be euclidean Einstein manifolds. In even dimensions, spacetimes with transverse section of non-constant curvature exist only in $d = 8$ and 10. This classification goes beyond standard supergravity and the eleven-dimensional case is analyzed. It is shown that if the transverse section has negative scalar curvature, only extended objects can have a supersymmetric ground state. In that case, some solutions are explicitly found whose ground state resembles a wormhole.

Keywords: Black Holes in String Theory, Black Holes.

JHEP05(2002)020

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Contents

1. Introduction

The presence of a negative cosmological constant allows the existence of black holes with topologically non-trivial transverse sections [1, 2, 3]. The features of these geometries have been extensively studied, see e.g. [4]–[9]. The simplest solution of the Einstein equations with negative cosmological constant of this kind in $d \geq 4$ dimensions, is described by the line element[1]

$$ds^2 = -\left(\gamma + \frac{r^2}{l^2} - \frac{\mu}{r^{d-3}}\right) dt^2 + \frac{dr^2}{\left(\gamma + \frac{r^2}{l^2} - \frac{\mu}{r^{d-3}}\right)} + r^2 d\sigma_\gamma^2\,, \tag{1.1}$$

where the constant μ is proportional to the mass. Here $d\sigma_\gamma^2 = \hat{g}_{ij}(y)dy^i dy^j$ is the line element of the $(d-2)$-dimensional transverse section Σ_γ, which is a Einstein manifold of euclidean signature,

$$\hat{R}_{ij} = \gamma(d-3)\hat{g}_{ij}\,. \tag{1.2}$$

The constant γ has been normalized to $\pm 1, 0$ by a suitable coordinate rescaling.

The electrically and magnetically charged extensions of (1.1) are also known [3]. The existence of an event horizon is ensured if Σ_γ is a compact and orientable surface. The Schwarzschild-AdS geometry is recovered when the transverse section is the unit sphere $(\gamma = 1)$.

Note that the configurations (1.1) are asymptotically locally AdS spacetimes only if the transverse section Σ_γ has constant curvature, namely, the curvature two-form[2] satisfies $\hat{R}^{mn} = \gamma \hat{e}^m \wedge \hat{e}^n$. In particular, this means that Σ_γ is locally isomorphic to the sphere S^n,

[1] The cosmological constant is given by $\Lambda = -l^{-2}(d-1)(d-2)/2$.

[2] This condition is automatically satisfied in four and five dimensions.

the hyperbolic manifold H^n, or the euclidean space $\mathbb{R}^n$. Furthermore, the Killing-Hopf theorem states that any n-dimensional complete connected Riemannian manifold of euclidean signature and constant curvature γ (for $n \geq 2$), has one of the following forms (see e.g. [10])

$$\begin{array}{lll} \dfrac{S^n}{\Gamma}\,, & \text{with } \Gamma \subset \mathrm{O}(n+1)\,, & \text{if } \gamma > 0\,, \\ \dfrac{H^n}{\Gamma}\,, & \text{with } \Gamma \subset \mathrm{O}(n,1)\,, & \text{if } \gamma < 0\,, \\ \dfrac{\mathbb{R}^n}{\Gamma}\,, & \text{with } \Gamma \subset \mathrm{ISO}(n)\,, & \text{if } \gamma = 0\,, \end{array}$$

where Γ is a freely acting discrete subgroup (i.e. without fixed points).

Asymptotically locally anti de Sitter black holes are non-trivial examples for testing the AdS/CFT correspondence [11]. Their asymptotic regions provide inequivalent background spacetimes where the corresponding dual thermal CFT is realized [4, 7, 12]. Thus, CFT's defined on $S^1 \times (\mathbb{R}^{d-2}/\Gamma)$, $S^1 \times (S^{d-2}/\Gamma)$ or $S^1 \times (H^{d-2}/\Gamma)$ are connected with black holes in the bulk for $\gamma = 0$, 1, or -1, respectively. Black holes with topologically non-trivial transverse sections exist also for gravitation theories containing higher powers of the curvature. Their relationship with thermal CFT has also been explored [9].

Since we assume a non-vanishing cosmological constant, and the geometry of Σ_γ is not necessarily spherical, these solutions do not satisfy the hypotheses of the standard positivity energy theorems for gravity and supergravity [13, 14, 15]. As a step towards a proof of an energy bound, we analyze the existence of supersymmetric ground states. This requirement not only ensures the stability of the ground state, but also impose severe restrictions on the geometry of Σ_γ, allowing a classification of the ground states.

The classification of euclidean manifolds admitting Killing spinors is obtained from their transformation properties under parallel transport along a closed loop, that is, from the holonomy group of the manifold. The possible holonomy groups, in turn, were classified by Berger [16].

In the following, the conditions for the family of spacetimes of the form (1.1) to admit Killing spinors is addressed. In section 2, the Killing spinors of spacetime are explicitly obtained, and they are completely determined by the Killing spinors of the transverse section. This occurs for $\mu = 0$ only.

In section 3, the supersymmetric ground states are analyzed for the different values of γ. Since a manifold of positive scalar curvature ($\gamma = 1$) is necessarily compact, the asymptotic region of non-rotating localized distributions of matter with a supersymmetric ground state are classified.

For $\gamma = -1$ supersymmetry requires the transverse section to be non compact. All supersymmetric geometries of the form (1.1) with non-compact transverse section of negative scalar curvature are classified. The transverse sections of these geometries contain a Ricci flat submanifold which determines the number of supersymmetries of spacetime. Warped black brane solutions are found whose supersymmetric ground state resembles a wormhole.

For $\gamma = 0$, the study is restricted to a particular class of Ricci flat transverse sections allowing the existence of proper black objects, which are classified demanding the existence a supersymmetric ground state.

JHEP05(2002)020

In section 4 it is shown that this classification of supersymmetric ground states in standard supergravity also applies to eleven-dimensional AdS Supergravity [17, 18].

2. Killing spinors and the transverse section

The spacetimes (1.1) can be viewed as solutions of standard supergravity theories with negative cosmological constant (see e.g. [19]). These configurations are left invariant under supersymmetry transformations, $\delta\psi = \nabla\epsilon$, provided ϵ is a global solution of the Killing spinor equation

$$\nabla\epsilon := (d + A)\epsilon = 0 \,, \tag{2.1}$$

where

$$A = \frac{1}{4}\omega^{ab}\Gamma_{ab} + \frac{1}{2l}e^a\Gamma_a \,, \tag{2.2}$$

and l is the AdS radius. The one-form A can be regarded as a connection for the AdS group $\mathrm{SO}(d-1,2)$, whose generators are expressed in terms of Dirac matrices as $J_{ab} = \frac{1}{2}\Gamma_{ab}$ and $J_a = \frac{1}{2}\Gamma_a$, and whose curvature $F = dA + A \wedge A$ is [3]

$$F = \frac{1}{2}\left(R^{ab} + \frac{1}{l^2}e^a \wedge e^b\right) J_{ab} + \frac{1}{l}T^a J_a \,. \tag{2.3}$$

The integrability condition for eq. (2.1) reads

$$\nabla\nabla\epsilon = F\epsilon = 0 \,. \tag{2.4}$$

As the torsion vanishes, the curvature for the metric (1.1) is

$$R^{ab} + \frac{1}{l^2}e^a \wedge e^b = \begin{cases} \mu\frac{(d-2)(d-3)}{2r^{d-1}}e^0 \wedge e^1 \,, \\ -\mu\frac{(d-3)}{2r^{d-1}}e^0 \wedge e^m \,, \\ -\mu\frac{(d-3)}{2r^{d-1}}e^1 \wedge e^m \,, \\ \hat{R}^{mn} - \gamma\hat{e}^m \wedge \hat{e}^n + \mu\frac{1}{r^{d-1}}e^m \wedge e^n \,. \end{cases}$$

The integrability condition (2.4) is satisfied only if $\mu = 0$ and

$$(\hat{R}^{mn} - \gamma\hat{e}^m \wedge \hat{e}^n)\Gamma_{mn}\epsilon = 0 \,.$$

Thus, the existence of Killing spinors shall be investigated for massless geometries whose line element is of the form

$$ds^2 = -(\gamma + r^2/l^2)dt^2 + \frac{dr^2}{(\gamma + r^2/l^2)} + r^2 d\sigma_\gamma^2 \,. \tag{2.5}$$

Choosing the frame as

$$e^0 = f(r)dt \,, \qquad e^1 = f(r)^{-1}dr \,, \qquad e^m = r\hat{e}^m \,,$$

where $f(r) = \sqrt{r^2/l^2 + \gamma}$, and $\hat{e}^m(y)$ is the vielbein of the transverse section Σ_γ, the

[3] Here ω^{ab} is the Lorentz connection one-form, e^a is the vielbein, $R^{ab} = d\omega^{ab} + \omega^a_{\ c} \wedge \omega^{cb}$ is the curvature two-form and $T^a = de^a + \omega^a_{\ b} \wedge e^b$ is the torsion.

JHEP05(2002)020

connection one-form A reads

$$A = \left(\frac{1}{2l}f(r) - \frac{r}{2l^2}\Gamma_1\right)\Gamma_0 dt + \frac{dr}{2lf(r)}\Gamma_1 + \frac{1}{4}\hat{\omega}^{mn}\Gamma_{mn} + \left(\frac{r}{l} - f(r)\Gamma_1\right)\frac{1}{2}\hat{e}^m\Gamma_m\,,$$

where $\hat{\omega}^{mn}(y)$ is the spin connection of the transverse section. In this frame, the solution of the Killing spinor equation (2.1) reads

$$\epsilon = e^{-\frac{\Gamma_1}{2}\ln\left(r/l+\sqrt{\gamma+r^2/l^2}\right)} e^{-\frac{t}{l}P_0}\eta\,, \tag{2.6}$$

where $\eta(y)$ satisfies

$$\left(d + \hat{A}_\gamma\right)\eta = 0\,, \tag{2.7}$$

and

$$\hat{A}_\gamma = \frac{1}{2}\hat{\omega}^{mn}J_{mn} + \hat{e}^m P_m\,. \tag{2.8}$$

Here

$$\begin{aligned} P_m &= \frac{1}{2}(P_- - \gamma P_+)\Gamma_m\,,\\ P_0 &= \frac{1}{2}\left(P_- + \gamma P_+\right)\Gamma_0\,, \end{aligned} \tag{2.9}$$

with $P_\pm := \frac{1}{2}(1 \pm \Gamma_1)$.

Note that since $[P_m, P_n] = -\gamma J_{mn}$, the set $\{P_n, J_{mn}\}$ forms a *reducible* representation for SO$(d-1)$, SO$(d-2,1)$, or ISO$(d-2)$ depending on whether $\gamma = 1, -1$, or 0, respectively. It is simple to express η in terms of irreducible $(d-2)$-dimensional spinors for each case (see below).

In this way, the problem of finding a Killing spinor ϵ for the spacetime eq. (2.5), has been reduced to that of finding a globally defined spinor η on the transverse section. Thus, one can formulate the following lemma

Lemma 1. *Killing spinors for the geometries (1.1) exist only for $\mu = 0$, and are completely determined by the Killing spinors of the transverse section.*

This situation is analogous to the case of conifold geometries, where the Killing spinors of an Einstein manifold X of positive scalar curvature are related to the Killing spinors of the cone over X, which is a Ricci flat manifold (see [20]). Here, the Killing spinors of the transverse section Σ_γ, which is an Einstein manifold of scalar curvature $\hat{R} = \gamma(d-2)(d-3)$, determine the Killing spinors of the spacetime (2.5), which is an Einstein manifold of negative scalar curvature.

Since complete, connected, irreducible Riemannian manifolds of euclidean signature admitting Killing spinors, have been classified [21, 22, 23] the above lemma allows the classification of the black objects (1.1) with a supersymmetric ground state. This is discussed in the following section.

JHEP05(2002)020

3. Classification of ground states

Since spinors may transform nontrivially under parallel transport along a closed loop, the maximal number of supersymmetries of a euclidean manifold is determined by its holonomy group [21], which are classified by Berger's theorem.

Killing spinors of a simply connected, complete and irreducible Einstein manifold X of positive scalar curvature were classified in [22] by using the conifold mapping between X and the cone over X which is Ricci flat. Analogously, there is a one-to-one correspondence between Killing spinors of an Einstein manifold of negative scalar curvature, Σ_{-1}, with the supersymmetries of Ricci flat manifold one dimension below [23].

Smooth manifolds can be obtained by making quotients of a symmetric space by discrete subgroups without fixed points. These quotients, in general, make the geometry non-simply connected and can introduce non-contractible loops which might further reduce the number of supersymmetries.

If the transverse section is a Ricci flat reducible space, then eq. (2.7) decomposes into the Killing spinor equation for each of its irreducible factors. Hence, Σ_0 admits global.

In what follows, the three cases $\gamma = \pm 1, 0$, are analyzed separately.

3.1 Positive curvature transverse section

In order to express η in terms of its irreducible parts, it is useful to introduce the projectors $\mathcal{Q}_{\pm} = (1/2 \pm iP_0)$ which commute with the one-form $\hat{A}$ defined by eq. (2.8). This allows splitting eq. (2.7) as

$$d\eta_{\pm} + \hat{A}_{\pm}\eta_{\pm} = 0\,, \tag{3.1}$$

where $\hat{A}_{\pm} = \mathcal{Q}_{\pm}\hat{A}$ are irreducible representations of $\hat{A}$, acting on $\eta_{\pm} = \mathcal{Q}_{\pm}\eta$, which are genuine spinors of the transverse section. When the dimension of Σ_1 is odd, then both representations are inequivalent, unlike the even-dimensional case.[4] Choosing the following representation for the Dirac matrices:[5] $\Gamma_0 = i\sigma_z \otimes \mathbb{I}$, $\Gamma_1 = \sigma_x \otimes \mathbb{I}$, $\Gamma_m = \sigma_y \otimes \gamma_m$, eq. (3.1) reads

$$d\eta_{\pm} + \left(\pm\frac{i}{2}\gamma_m \hat{e}^m + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\right)\eta_{\pm} = 0\,, \tag{3.2}$$

which means that $\eta_{\pm}$ must be a globally defined Killing spinor on Σ_1. Let $N_{\pm}$ be the maximum possible number of solutions of type $\eta_{\pm}$. Then, using the classification of positive scalar curvature euclidean manifolds admitting Killing spinors in [22], one can formulate the following theorem:

Theorem 1. *Let $\mathcal{M}$ be a d-dimensional manifold of the form (1.1) with $\gamma = 1$, whose transverse section Σ_1 is a simply connected, complete and irreducible Riemannian manifold of positive scalar curvature. If $\mathcal{M}$ possesses a supersymmetric ground state, then $\mathcal{M}$ can be either*

[4] When Σ_1 is $2n$-dimensional, then $-\gamma^i = \gamma_{2n+1}\gamma^i\gamma_{2n+1}$, with $\gamma_{2n+1} = (i)^n\gamma_1\gamma_2\cdots\gamma_{2n}$.

[5] Here and henceforth, σ_i are the Pauli matrices and γ_m satisfy $(d-2)$-dimensional Clifford algebra, $\{\gamma_m, \gamma_n\} = 2\delta_{mn}$, with $m = 2, \ldots, d-1$.

JHEP05(2002)020

(i) *a d-dimensional Schwarzschild-AdS spacetime, whose ground state is AdS, admitting the maximum number of supersymmetries, namely* $2^{[d/2]}$,

(ii) *an eight-dimensional black hole whose transverse section is a nearly Kähler manifold, and its ground state admits one Killing spinor of each type ($N_+ = N_- = 1$), or*

(iii) *an odd-dimensional black hole with $d \geq 7$, whose transverse section geometry and the corresponding maximum number of Killing spinors of its ground state are given by the following table,*

d	Σ_1	(N_+, N_-)
7	*Sasaki-Einstein*	$(1,1)$
9	*3-Sasaki*	$(3,0)$
	Sasaki-Einstein	$(2,0)$
	Nearly parallel G_2	$(1,0)$
11	*Sasaki-Einstein*	$(1,1)$

Table 1.

Since, under the assumptions of the above theorem, the transverse section Σ_1 is necessarily a compact manifold, this theorem classifies the asymptotic region of localized non-rotating distributions of matter with a supersymmetric ground state.

From a mathematical point of view, the Killing spinor equation can be solved regardless the existence of a supergravity theory. Indeed, the transverse section for $d = 4k - 1 > 11$ dimensions, can be a Sasaki-Einstein manifold admitting $(1,1)$ Killing spinors; and for $d = 4k + 1 > 9$ the surface Σ_1 can be either a Sasaki-Einstein or a 3-Sasaki manifold admitting $(2,0)$ and $(k+1,0)$ Killing spinors, respectively.

As mentioned above, performing identifications on these transverse sections, break some supersymmetries in general. For instance, consider the smooth quotients of the sphere which have been fully classified (see e.g. [10]). In even dimensions, real projective spaces — that is, the sphere with antipodal points identified, $\mathbb{RP}^{2n} = S^{2n}/\mathbb{Z}_2$ — are the only possible smooth quotients. However, these are not orientable manifolds and therefore they cannot correspond to the event horizon of a black hole.

In odd dimensions, smooth quotients of the form $\Sigma_1 = S^{2n-1}/\Gamma$ are always orientable, and among them, there are some interesting cases with unbroken supersymmetries (see e.g. ref. [24]). These correspond to the transverse section of topological black holes whose ground states are locally AdS spaces with unbroken supersymmetries. For instance, the real projective space $\mathbb{RP}^{2n-1}$ admits $(2^{n-1},0)$ Killing spinors, provided n is even [22]. Thus, in nine dimensions, if $\Sigma_1 = \mathbb{RP}^7$, the spacetime admits 8 Killing spinors which are eigenstates of $\mathcal{Q}_+$ and zero eigenstates of $\mathcal{Q}_-$ (or vice-versa).

3.2 Negative curvature transverse section

The line element (1.1) can be viewed as the exterior geometry of a localized non-rotating distribution of matter, provided the transverse section Σ_γ is a compact euclidean Einstein manifold (see eq. (1.2)). However, as can be easily seen, a compact transverse section

Σ_γ admits no Killing spinors for $\gamma = -1$. Consider vector ξ^m which satisfies the Killing equation on Σ_{-1}. Then, the following identity holds

$$\int_{\Sigma_{-1}} \nabla_m \xi_n \nabla^m \xi^n \sqrt{\hat{g}} d^{d-2}x = \gamma(d-3) \int_{\Sigma_{-1}} \xi_m \xi^m \sqrt{\hat{g}} d^{d-2}x \,. \tag{3.3}$$

Since Σ_γ has euclidean signature, the left-hand side of eq. (3.3) is non negative, and therefore, for $\gamma = -1$, ξ^m necessarily vanishes. Furthermore, since for any Killing spinor η, the vector field $\xi^m := \bar{\eta}\gamma^m\eta$ would be an isometry, one concludes that any compact euclidean Einstein manifold with negative scalar curvature cannot have either Killing vectors, or Killing spinors.

As a consequence of this, and by virtue of the lemma in section 2, black holes with compact transverse sections cannot be BPS states regardless the value of the mass. In particular, in this case, the solution with $\mu = 0$ describes a black hole with horizon radius $r_+ = l$ and temperature $\beta = 2\pi l$. Now, since the specific heat is positive, this state could decay by Hawking radiation into black holes with $0 > \mu \geq \mu_c = -\frac{l^{d-3}}{G}\sqrt{(d-3)^{d-3}/(d-1)^{d-1}}$ [9].

Therefore, demanding that the metric (2.5) admit globally defined Killing spinors, leads one to consider geometries with non-compact transverse sections Σ_{-1}. Non-compact Riemannian manifolds, which are complete, connected and irreducible, of euclidean signature and negative scalar curvature, admitting Killing spinors have been classified [23]. Since the line element is isometric to

$$d\sigma_{-1}^2 = \frac{1}{z^2}(dz^2 + h_{ij}dx^i dx^j) \,, \tag{3.4}$$

where h_{ij} is the metric of a $d-3$-dimensional complete connected Ricci-flat manifold admitting Killing spinors, the classification of these spaces is obtained from the classification of Ricci flat manifolds in ref. [21].

In complete analogy with case $\gamma = 1$, the metrics of the form (2.5) with $\gamma = -1$ admit Killing spinors given by eq. (2.6). The irreducible components of $\hat{A}$ and η are also given by $\hat{A}_\pm = \mathcal{Q}_\pm \hat{A}$ and $\eta_\pm = \mathcal{Q}_\pm \eta$, respectively,[6] where now $\mathcal{Q}_\pm := (\frac{1}{2} \pm P_0)$.

Using the representation for the Dirac matrices in which $\Gamma_0 = i\sigma_y \otimes \mathbb{I}$, $\Gamma_1 = \sigma_x \otimes \mathbb{I}$, $\Gamma_m = \sigma_z \otimes \gamma_m$ for $\gamma = -1$, one finds that $\eta_\pm$ satisfy,

$$d\eta_\pm + \left(\pm\frac{1}{2}\gamma_m \hat{e}^m + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\right)\eta_\pm = 0 \,. \tag{3.5}$$

Let $N_\pm$ be the maximum possible number of solutions of type $\eta_\pm$. Then, since negative scalar curvature euclidean manifolds admitting Killing spinors are classified in [23], the following theorem holds:

Theorem 2. *Let $\mathcal{M}_0$ be a d-dimensional manifold of the form (2.5) with $\gamma = -1$, admitting Killing spinors, whose transverse section Σ_{-1} is a non-compact, connected, complete and irreducible manifold of negative scalar curvature. Let Ξ be the complete, connected Ricci-flat submanifold described by h_{ij} in eq. (3.4). If $\mathcal{M}_0$ possesses a supersymmetric ground state, then $\mathcal{M}_0$ can be either*

[6] When the dimension of Σ_{-1} is even both representations are equivalent, unlike the odd case.

JHEP05(2002)020

(i) *a d-dimensional portion of AdS whose transverse section is* H^{d-2}*, which admits* $2^{[(d-2)/2]}$ *Killing spinors,*

(ii) *a ten-dimensional manifold where* Ξ *is a seven-dimensional space with* G_2 *holonomy admitting only one Killing spinor, or*

(iii) *an odd-dimensional manifold with* $d \geq 7$*, where the geometry, holonomy and corresponding maximal number of Killing spinors of* Ξ *is given by the following table,*

d	Ξ	*Hol*(Ξ)	(N_+, N_-)
7	*hyperkähler*	Sp(2)	$(2,0)$
	Calabi-Yau	SU(2)	$(2,0)$
9	*Calabi-Yau*	SU(3)	$(1,1)$
11	*hyperkähler*	Sp(4)	$(3,0)$
	Calabi-Yau	SU(4)	$(2,0)$
	Parallel Spin_7	Spin(7)	$(1,0)$

Table 2.

As in the previous case, independently from the existence of supergravity, for $d = 4k+1 > 9$, the surface Ξ can be a Calabi-Yau manifold with $\mathrm{SU}(2k-1)$ holonomy which admits $(1,1)$ Killing spinors, while for $d = 4k-1 > 11$, Ξ can be either a Calabi-Yau manifold with $\mathrm{SU}(2k-2)$ holonomy admitting $(2,0)$ Killing spinors, or hyperkähler with $\mathrm{Sp}(2k-2)$ holonomy which admits $(k,0)$ Killing spinors.

3.2.1 Supersymmetry and extended objects

Remarkably, for $\gamma = -1$, the requirement of supersymmetry implies non compactness of the transverse section Σ_{-1}. This in turn means that only extended objects can have a supersymmetric ground state.

Let us consider for example, the four-dimensional case, whose transverse section Σ_{-1} is a two-dimensional surface of negative constant curvature. If $\Sigma_{-1} = H_2$, the metric (2.5) describes a portion of AdS_4 instead of a topological black hole [2, 3]. However, if one considers a quotient of the form $\Sigma_{-1} = H_2/\Gamma$, which is topologically a cylinder ($\mathbb{R} \times S^1$), then the metric (1.1) with $\gamma = -1$ describes a warped black string which may, or may not, possess a supersymmetric ground state, depending on Γ, as it is seen in the following examples

∘ *Non-supersymmetric "ground state"*

Consider Σ_{-1} described by the metric,

$$d\sigma_{-1}^2 = d\zeta^2 + \cosh^2\zeta d\varphi^2 \,, \tag{3.6}$$

with $-\infty < \zeta < \infty$ and $0 < \varphi \leq \alpha$, obtained by an identification on H_2 along the boost $\Gamma = \alpha\partial_\phi$. The solution of the Killing spinor equation (3.5) is given by

$$\eta = \exp\left(-\frac{\zeta}{2}\sigma_2\right)\exp\left(-\frac{\varphi}{2}\sigma_3\right)\eta_0 \,. \tag{3.7}$$

However, η is not globally defined because $\eta(\zeta,\alpha) \neq \pm\eta(\zeta,0)$, and therefore this space admits no Killing spinors. For $\mu = 0$ the spacetime is locally AdS and has an event horizon at $r_+ = l$. This solution is not supersymmetric and could decay by Hawking radiation to a state with $\mu < 0$. Cosmic censorship requires the ground state to be the solution with $\mu_c = -l/3\sqrt{3}G$, which is not supersymmetric either.

∘ *Supersymmetric ground state*

Consider Σ_{-1} to be the Poincaré upper half *cylinder*

$$d\sigma_{-1}^2 = \frac{1}{z^2}\left(dz^2 + d\varphi^2\right), \tag{3.8}$$

with $0 < z < \infty$ and $0 < \varphi \leq \alpha$, which is obtained by wrapping H_2 along the isometry $\Gamma = \alpha\partial_\varphi$. Then the solution of the Killing equation is

$$\eta(z,\varphi) = \exp\left(-\frac{\ln(z)}{2}\sigma_2\right)\left(I - \frac{\varphi}{2}\sigma_3(I+\sigma_2)\right)\eta_0\,. \tag{3.9}$$

This is a globally defined spinor provided $(I+\sigma_2)\eta_0 = 0$. Thus, in four dimensions, the line element

$$ds^2 = -\left(r^2/l^2 - \frac{\mu}{r} - 1\right)dt^2 + \frac{dr^2}{\left(r^2/l^2 - \frac{\mu}{r} - 1\right)} + \frac{r^2}{z^2}(dz^2 + d\varphi^2)\,, \tag{3.10}$$

describes a warped black string of mass $M = V_2\frac{\mu}{8\pi}$, where V_2 is the area of the transverse section.

For $\mu = 0$ the metric (3.10) has an event horizon at $r_+ = l$ with temperature $\beta = 2\pi l$. This suggests that it could evaporate decaying by Hawking radiation into states with negative mass. The possibility of decay, however, would be in conflict with the fact that the massless state is supersymmetric.

A supersymmetric ground state with metric (3.10) is given by

$$ds^2 = -\sinh^2\left(\frac{w}{l}\right)dt^2 + dw^2 + l^2\cosh^2\left(\frac{w}{l}\right)\frac{dz^2 + d\varphi^2}{z^2}\,, \tag{3.11}$$

with $-\infty < w < \infty$. This manifold $\mathcal{M}$ has negative constant curvature and is smooth everywhere. It possesses a horizon at $w = 0$, where an Einstein-Rosen bridge is centered, and its boundary is formed by two connected components, defined by $w = \pm\infty$, so that $\partial\mathcal{M}$ is a manifold of negative scalar curvature. This does not contradict the no-go theorem for wormholes in AdS of ref. [25], which states that a boundary with positive scalar curvature must be connected.

In this scenario an observer sees a *warped black string* with $\mu > 0$ decaying towards a supersymmetric final state (3.11) preserving half of the supersymmetries of AdS_4, generated by the Killing spinors

$$\epsilon = \exp\left(-\frac{w}{2l}\Gamma_1\right)\exp\left(\frac{t}{2l}\Gamma_1\Gamma_0\right)\eta\,, \tag{3.12}$$

with

$$\eta = \begin{pmatrix} \exp\left(-\frac{\ln(z)}{2}\sigma_2\right)\xi_+ \\ \exp\left(\frac{\ln(z)}{2}\sigma_2\right)\xi_- \end{pmatrix},$$

where the constant two-dimensional spinors $\xi_\pm$ satisfy $\sigma_2\xi_\pm = \pm\xi_\pm$. This ground state can be obtained from the exterior metric of (3.10) with $\mu = 0$, $r = l\cosh(w/l)$, and thereafter continuing w to negative values.

Warped black branes of higher dimensions, analogous to (3.10), can be readily found,

$$ds^2 = -\left(r^2/l^2 - \frac{\mu}{r^{d-3}} - 1\right)dt^2 + \frac{dr^2}{\left(r^2/l^2 - \frac{\mu}{r^{d-3}} - 1\right)} + r^2 d\sigma_{-1}\,, \tag{3.13}$$

where the $d-2$-dimensional transverse sections Σ_{-1} have metric

$$d\sigma_{-1}^2 = \frac{1}{z^2}(dz^2 + \delta_{ij}dx^i dx^j)\,, \tag{3.14}$$

with $0 < z < \infty$ and at least one of the x^i's is compact. The mass is given by $M = V_{d-2}\frac{\mu}{2\Omega_{d-2}}$ where Ω_{d-2} is the volume of $d-2$ sphere. The ground state is given by the higher-dimensional version of metric (3.11) with transverse section (3.14). In this case, the ground state preserves one half of the supersymmetries of AdS_d, and the Killing spinors of Σ_{-1} are

$$\eta_\pm = \exp\left(\mp\frac{\ln(z)}{2}\gamma_2\right)\xi_\pm\,, \tag{3.15}$$

where $(I \pm \gamma_2)\xi_\pm = 0$. This last condition comes from the periodicity in one of the x^i's. Note that these Killing spinors depend only on z, and therefore the coordinates x^i can be further wrapped without breaking additional supersymmetries.[7]

3.3 Ricci flat transverse section

For $\gamma = 0$, the solution of the Killing spinor equation (2.6) reads

$$\epsilon = \exp\left(-\frac{\Gamma_1}{2}\ln\left(\frac{r}{l}\right)\right)\left(1 - \frac{t}{l}P_0\right)\eta\,, \tag{3.16}$$

where η satisfies $(d + \hat{A})\eta = 0$. Here $\hat{A} = \frac{1}{2}\hat{\omega}^{mn}J_{mn} + \hat{e}^m P_m$, where J_{mn} and P_m generate ISO$(d-2)$, and P_0, P_m can be read from eq. (2.9). Due to the presence of the local translation generators P_m, the spinor η does not necessarily satisfy the standard Killing spinor equation,

$$d\eta + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\eta = 0\,. \tag{3.17}$$

The consistency conditions, however, still require the transverse section Σ_0 to be Ricci flat. Moreover, the integration constant μ could be rescaled away in the metric (1.1) with $\gamma = 0$, unless one of the coordinates of the transverse section y^i is compact. In order to avoid this

[7] The Killing spinors of H^{d-2}, written in terms of Poincaré coordinates, are

$$\eta = \left(e^{-\frac{1}{2}\ln(z)\gamma_2}\prod_{i=3}^{d-1}\left(1 - \frac{1}{2}x^i\gamma_i(I + \gamma_2)\right)\right)\eta_0.$$

For the transverse section (3.14) the compactness of a single x^i implies the projection condition $(I + \gamma_2)\eta_0 = 0$, and additional wrappings along the other directions do not impose further constraints.

ambiguity, transverse geometries with one wrapped direction, of the form $\Sigma_0 = S^1 \times \Xi$, shall be considered. The line element of Σ_0 reads,

$$d\sigma_0^2 = d\phi^2 + h_{ij}(x)dx^i dx^j \,, \tag{3.18}$$

where h_{ij} is the metric of the $(d-3)$-dimensional Ricci flat euclidean space Ξ, and ϕ parametrizes S^1. In this case, the solution of the Killing equation is given by

$$\eta = \left(1 - \frac{1}{2}\phi\Gamma_2 P_+\right)\tilde{\eta}(x)\,,$$

where $\tilde{\eta}(x)$ satisfies the standard Killing equation on Ξ. The periodicity condition implies that $P_+\tilde{\eta} = 0$, which projects out half of the components of η, and thus, the spinor $\eta_- = P_-\eta$ satisfies

$$d\eta_- + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\eta_- = 0\,, \tag{3.19}$$

where the representation $\Gamma_0 = i\sigma_y \otimes \mathbb{I}, \Gamma_1 = \sigma_z \otimes \mathbb{I}, \Gamma_m = \sigma_x \otimes \gamma_m$ has been used. In this way, Killing spinors for the spacetime (2.5) with $\gamma = 0$ possess only one chirality. Now, since $\eta = \eta_-$ solves eq. (3.19), the supersymmetric Ricci flat surfaces Ξ are classified [21], therefore, black objects given by (1.1) with $\gamma = 0$, whose transverse sections have metric (3.18), possessing a supersymmetric ground state, can be classified according to the following theorem:

Theorem 3. *Let $\mathcal{M}$ be a d-dimensional manifold of the form (1.1) with $\gamma = 0$, with transverse section of the form $\Sigma_0 = S^1 \times \Xi$, where Ξ is a simply connected, complete and irreducible Ricci flat manifold. If $\mathcal{M}$ possesses a globally supersymmetric ground state, then it can be either*

(i) *a d-dimensional black brane with $\Sigma_0 = S^1 \times \mathbb{R}^{d-3}$, whose ground state ($\mu = 0$), is a locally AdS spacetime admitting $2^{[(d-3)/2]}$ Killing spinors,*

(ii) *a ten-dimensional black object, where Ξ is a seven-dimensional space with G_2 holonomy admitting only one Killing spinor, or*

(iii) *an odd-dimensional black object with $d \geq 7$, where the geometry, holonomy and corresponding maximal number of Killing spinors of Ξ are the same as those listed in table 2.*

In higher dimensions, the number of Killing spinors, as well as the geometry of Ξ can be readily obtained from those in the previous section. Note that if Ξ is assumed to be non-simply connected, then in the maximally supersymmetric case, where $\Sigma_0 = S^1 \times \mathbb{R}^{2n-3}$, the remaining coordinates can be further wrapped, so that $\Sigma_0 = (S^1)^{p+1} \times \mathbb{R}^{2n-3-p}$ with $0 \leq p \leq 2n-3$, without breaking additional supersymmetries.

4. AdS Supergravity in eleven dimensions

It has been shown that standard eleven-dimensional supergravity [26] cannot accommodate a cosmological constant [27], so it would be interesting to examine whether the supersymmetric solutions discussed here make sense in eleven dimensions. It turns out that these solutions are BPS states of eleven-dimensional AdS supergravity [17, 28]. The field content of the $\mathcal{N} = 1$ theory is the graviton e_μ^a, a gravitino ψ_μ, the spin connection ω_μ^{ab}, and a bosonic 1-form b_μ^{abcde} which is an antisymmetric fifth-rank Lorentz tensor in tangent space. These fields form a connection for the super AdS_{11} group, OSp(32|1), whose algebra is expected to be the underlying M-Theory symmetry. The action describes a gauge theory with a fiber bundle structure, and the lagrangian is a Chern-Simons density. The purely gravitational sector of the lagrangian contains a negative cosmological constant and the Einstein-Hilbert term, plus some additional terms with higher powers of the Riemann curvature, combined in such a way as to yield second order field equations for the metric.

This theory possesses solutions of the form [9]

$$ds^2 = -(\gamma + r^2/l^2 - (2G\mu)^{1/5})dt^2 + \frac{dr^2}{(\gamma + r^2/l^2 - (2G\mu)^{1/5})} + r^2 d\sigma_\gamma^2\,, \tag{4.1}$$

where the integration constant μ is related to the mass through $\mu = \frac{\Omega_9}{V_9}M + \frac{1}{2G}\delta_{1,\gamma}$. These configurations are left invariant under the supersymmetry transformation $\delta\psi = \nabla\epsilon$, provided ϵ solve the same the Killing spinor equation as eq. (2.1) and, as a consequence, requiring supersymmetry restricts the transverse section to be an Einstein manifold of scalar curvature $\hat{R} = 72\gamma$. With this last condition, the metric (4.1) solves the field equations, even though they are not those of Einstein's theory.

The supersymmetric ground states correspond to the metric (4.1) with $\mu = 0$, which for $d = 11$ is the same as eq. (2.5), and therefore, the classification in eleven dimensions can be obtained from the theorems above. If $\gamma = 1$, the transverse section can be either S^9 or a Sasaki-Einstein manifold. If $\gamma = -1$, the transverse section is a negative scalar curvature manifold of the form (3.9) with a subsection Ξ which can be either $\mathbb{R}^8$, hyperkähler, Calabi-Yau, or a Parallel Spin$_7$ manifold. Finally, if $\gamma = 0$ and the transverse section is $\Sigma_0 = S^1 \times \Xi$, then the surface Ξ coincides with the previous case.

5. Discussion

Black objects of the form (1.1) possessing a supersymmetric ground state have been classified. These geometries necessarily have a constant curvature transverse section Σ, if the spacetime dimension is less than seven. For $d \geq 7$, the transverse section Σ can be also any of the euclidean Einstein manifolds listed in tables 1 and 2. Since the existence of manifolds with exceptional holonomy, even-dimensional spacetimes with a non-constant curvature transverse section exist only for $d = 8$, being Σ a surface of positive scalar curvature, and for $d = 10$, with Σ a non-positive scalar curvature manifold.

In odd dimensions, this classification goes beyond standard supergravity, in particular, the eleven-dimensional case was analyzed in section 4.

As it occurs for conifold geometries one would expect that besides the mapping of Killing spinors between a supersymmetric ground state and its transverse section, further structures can be connected. Following this scheme, one would expect that other BPS states as branes or product spaces can be classified.

The spacetimes discussed here are asymptotically locally AdS, only when the transverse section has constant curvature, such as the massless configuration (2.5) is a constant curvature manifold. In this case, the curvature $F = dA + A \wedge A$ vanishes, so that the AdS connection A in eq. (2.2) can be expressed in terms of an element of SO$(d-1,2)$ as $A = g^{-1}dg$. Hence, the Killing spinor equation is solved by

$$\epsilon = g^{-1}\epsilon_0\,, \tag{5.1}$$

where ϵ_0 is a constant spinor, and the group element reads

$$g(t,r,y^m) = T(y^m)e^{\frac{t}{l}P_0}e^{\frac{\Gamma_1}{2}\ln(r/l+\sqrt{\gamma+r^2/l^2})}\,, \tag{5.2}$$

with $T(y^m)$ satisfying

$$dT(y^m) = T(y^m)\hat{A}_\gamma\,. \tag{5.3}$$

Here $\hat{A}$ is a flat connection for the transverse section, which means that the Killing spinors of Σ are given by $\eta = T^{-1}\epsilon_0$. In the case $\gamma = -1$, the temperature, $\beta = 2\pi l$, can be found either by demanding regularity of the euclidean solution at the horizon or by demanding the holonomy of A to be trivial, that is $g^{-1}(\beta,r,y)g(0,r,y) = 1$, where $g(\tau,r,y)$ is obtained from eq. (5.2) by a Wick rotation. Killing spinors for other locally AdS spacetimes have been discussed also [29]–[32].

For $\gamma = -1$ the requirement of global supersymmetry implies that only extended objects can have a supersymmetric ground state. An explicit example resembling a wormhole was constructed (see eqs. (3.10), (3.11), and (3.13)), which is a supersymmetric state with non-vanishing temperature as it occurs for some BPS branes in ref. [33].

If the transverse section were a Ricci flat manifold which differs from $\Sigma_0 = S^1 \times \Xi$, the classification deals with a different problem. In that case, for $\gamma = 0$, eq. (2.7) on the transverse section does not reduce in general to the standard Killing spinor equation. In fact, using the representation $\Gamma_0 = i\sigma_y \otimes \mathbb{I}, \Gamma_1 = \sigma_z \otimes \mathbb{I}, \Gamma_m = \sigma_x \otimes \gamma_m$, the spinors $\eta_\pm = P_\pm \eta$ satisfy

$$d\eta_+ + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\eta_+ = 0\,, \tag{5.4}$$

and

$$d\eta_- + \frac{1}{4}\hat{\omega}^{mn}\gamma_{mn}\eta_- = \frac{1}{2}\gamma_m\hat{e}^m\eta_+\,. \tag{5.5}$$

The solutions for η_- can be trivially written in terms of η_+. Indeed, the consistency condition for eq. (5.5) gives the same information as eq. (5.4), i.e. Σ_0 must be Ricci flat. However, equation (5.5) may be incompatible with some of the global properties of Σ_0, and hence it would in general restrict the possible spaces where eq. (5.4) has a global solution.

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References

[1] J.P.S. Lemos, *Cylindrical black hole in general relativity*, *Phys. Lett.* **B 353** (1995) 46 [gr-qc/9404041].

[2] L. Vanzo, *Black holes with unusual topology*, *Phys. Rev.* **D 56** (1997) 6475 [gr-qc/9705004].

[3] D.R. Brill, J. Louko and P. Peldan, *Thermodynamics of* $(3+1)$*-dimensional black holes with toroidal or higher genus horizons*, *Phys. Rev.* **D 56** (1997) 3600 [gr-qc/9705012].

[4] D. Birmingham, *Topological black holes in anti-de Sitter space*, *Class. and Quant. Grav.* **16** (1999) 1197 [hep-th/9808032].

[5] R.-G. Cai and K.-S. Soh, *Topological black holes in the dimensionally continued gravity*, *Phys. Rev.* **D 59** (1999) 044013 [gr-qc/9808067].

[6] M.M. Caldarelli and D. Klemm, *Supersymmetry of anti-de Sitter black holes*, *Nucl. Phys.* **B 545** (1999) 434 [hep-th/9808097].

[7] R. Emparan, C.V. Johnson and R.C. Myers, *Surface terms as counterterms in the AdS/CFT correspondence*, *Phys. Rev.* **D 60** (1999) 104001 [hep-th/9903238].

[8] G.J. Galloway, K. Schleich, D.M. Witt and E. Woolgar, *Topological censorship and higher genus black holes*, *Phys. Rev.* **D 60** (1999) 104039 [gr-qc/9902061].

[9] R. Aros, R. Troncoso and J. Zanelli, *Black holes with topologically nontrivial AdS asymptotics*, *Phys. Rev.* **D 63** (2001) 084015 [hep-th/0011097].

[10] J.A. Wolf, *Spaces of constant curvature*, ch. 2, p. 69. Publish or Perish, Wilmington, Delaware (USA), fifth ed., 1984.

[11] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[12] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[13] S. Deser and C. Teitelboim, *Supergravity has positive energy*, *Phys. Rev. Lett.* **39** (1977) 249.

[14] R. Schoen and S.-T. Yau, *On the proof of the positive mass conjecture in general relativity*, *Commun. Math. Phys.* **65** (1979) 45.

[15] E. Witten, *A simple proof of the positive energy theorem*, *Commun. Math. Phys.* **80** (1981) 381.

[16] M. Berger, *Sur les groupes d'holonomie des variétés à connexion affine et des variétés riemanniennes*, *Bull. Soc. Math. France* **83** (1955) 279; *Remarques sur le groupe d'holonomie des variétés riemanniennes*, *C. R. Acad. Sc. Paris.* **262** (1966) 1316;
See also, S. Weinberg, *Elementary differential geometry from a generalized standpoint* in proceedings, *Physics in higher dimensions*, Jerusalem 1984/85, pp. 160–203.

[17] R. Troncoso and J. Zanelli, *New gauge supergravity in seven and eleven dimensions*, *Phys. Rev.* **D 58** (1998) 101703 [hep-th/9710180].

[18] R. Troncoso and J. Zanelli, *Chern-Simons supergravities with off-shell local superalgebras*, in *Black holes and the structure of the universe*, C. Teitelboim and J. Zanelli eds., Santiago 1997, Singapore, World Scientific 2000, pp. 119–145 [hep-th/9902003].

JHEP05(2002)020

[19] *Supergravities in diverse dimensions*, A. Salam and E. Sezgin eds., North-Holland, World Scientific, Amsterdam, Netherlands 1989.

[20] J.M. Figueroa-O'Farrill, *On the supersymmetries of anti de Sitter vacua*, *Class. and Quant. Grav.* **16** (1999) 2043 [hep-th/9902066].

[21] M. Wang, *Parallel spinors and parallel forms*, *Ann. Global Anal. Geom.* **7** (1989) 59.

[22] C. Bär, *Real Killing spinors and holonomy*, *Comm. Math. Phys.* **154** (1993) 509.

[23] H. Baum, *Complete riemannian manifolds with imaginary Killing spinors*, *Ann. Global Anal. Geom.* **7** (1989) 205.

[24] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[25] E. Witten and S.T. Yau, *Connectedness of the boundary in the AdS/CFT correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1635 [hep-th/9910245].

[26] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[27] K. Bautier, S. Deser, M. Henneaux and D. Seminara, *No cosmological $D = 11$ supergravity*, *Phys. Lett.* **B 406** (1997) 49 [hep-th/9704131].

[28] R. Troncoso and J. Zanelli, *Gauge supergravities for all odd dimensions*, *Int. J. Theor. Phys.* **38** (1999) 1181 [hep-th/9807029].

[29] M. Bañados, A. Gomberoff and C. Martínez, *Anti-de Sitter space and black holes*, *Class. and Quant. Grav.* **15** (1998) 3575 [hep-th/9805087].

[30] B. Ghosh and S. Mukhi, *Killing spinors and supersymmetric AdS orbifolds*, *J. High Energy Phys.* **10** (1999) 021 [hep-th/9908192].

[31] A. Kehagias and J.G. Russo, *Hyperbolic spaces in string and M-theory*, *J. High Energy Phys.* **07** (2000) 027 [hep-th/0003281].

[32] E. Shuster, *Killing spinors and supersymmetry on AdS*, *Nucl. Phys.* **B 554** (1999) 198 [hep-th/9902129].

[33] A. Sen, *Extremal black holes and elementary string states*, *Mod. Phys. Lett.* **A 10** (1995) 2081 [hep-th/9504147].